

ROM2F-96/14
February 1, 2008

Open Superstrings¹

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Abstract

We review the basic principles of the construction of open and unoriented superstring models and analyze some representative examples.

¹Talk presented at the “IV Italian-Korean Meeting on Relativistic Astrophysics”, Rome - Gran Sasso - Pescara, July 9-15, 1995.

1. Introduction

(Super)String theories [1] have emerged in the last 20 years as the only candidate to describe in a unified scheme all the fundamental interactions, including gravity. Although closed oriented models are often thought to be more elegant and promising, we would like to convince the reader that models of closed and open unoriented (super)strings (called for simplicity open) own the same level of consistency. This is also suggested from the recent ideas on string dualities, according which all (super)string theories, being connected in a non-perturbative way, are different manifestations of an underlying, yet unknown, entity [2]. Perturbatively, open (super)strings and, say, heterotic strings [3] look very different, but there are evidences for non-perturbative dualities connecting them [4].

In this talk we shall review the fundamental aspects of the construction of open and unoriented (super)string models. In section 2, we briefly summarize how to define closed 2-D conformal field theories (CFT), the basic building blocks of closed (super)strings. In section 3 we define CFT on Riemann surfaces with holes and/or crosscaps and review our algorithm for associating a class of “open descendants” to left-right symmetric closed CFT using a “parameter space orbifold” construction [5]. In particular, we shall be able to describe the perturbative spectra of models with corresponding internal (Chan-Paton) symmetry groups and related patterns of symmetry breaking. Finally, in section 4, we analyze some significant examples.

2. Closed CFT and superstrings

The most important property of 2-D (closed) CFT [6] is the factorization of the stress-energy tensor into chiral components $T(z)$ and $\bar{T}(\bar{z})$, each a function of the single variable z , respectively \bar{z} . The two chiral sectors of the theory are almost independent and can be separately solved, provided that, at the end, \bar{z} is identified with the complex conjugate of z . This implies that the observable algebra splits into a tensor product of chiral algebras

$$\mathcal{A} \otimes \bar{\mathcal{A}} \tag{1}$$

each containing the stress-energy tensor and thus the Virasoro algebra. In the operator formalism, the decomposition (1) corresponds to the factorization of the Hilbert space as well. To be precise, conformal fields are organized in representations of the full symmetry algebra of the model. The space of states, however, is the superposition of irreducible representations (or, better, of superselection sectors):

$$\mathcal{H} = \bigoplus_{\Lambda, \bar{\Lambda}} \mathcal{H}_{\Lambda} \otimes \bar{\mathcal{H}}_{\bar{\Lambda}} \quad . \quad (2)$$

The sum in (2) is, in general, over an infinite number of sectors. Rational CFT [7] are characterized by the fact that the sum in (2) contains a finite number of terms. Bulk conformal fields are “intertwining operators” (not chiral) between different sectors, and can thus be decomposed in sums (finite in RCFT) of products of *chiral vertex operators* (CVO) [7] [8] [9]

$$\phi_{k, \bar{k}}(z, \bar{z}) = \sum_{i, \bar{i}, f, \bar{f}} V_k^f(z) \bar{V}_{\bar{k}}^{\bar{f}}(\bar{z}) \alpha^{i\bar{i}}_{f\bar{f}} \quad . \quad (3)$$

$V_k^f(z)$ denotes a field in the sector k of conformal dimension Δ_k acting on a state i and producing a state f , that can be non trivial only if f is in the fusion of i and k . In the simplest (diagonal) cases $\alpha^{i\bar{i}}_{f\bar{f}} = \delta^{i\bar{i}} \delta_{f\bar{f}}$. It should be noticed that CVO are multivalued functions of z , but the invariance of conformal fields under the transformation

$$U = e^{2\pi i (L_0 - \bar{L}_0)} \quad (4)$$

forces the conformal weights to obey the relation $\Delta_r - \bar{\Delta}_{\bar{r}} \in \mathbf{Z}$ for each representation r . Moreover CVO, exhibiting non trivial monodromies, satisfy a braid group statistics. They are not uniquely determined by eq. (3). On the contrary, the gauge freedom in the definition of CVO reflects itself in the nature of conformal fields as invariant tensors of a *quantum* symmetry [10].

We are in general interested in the calculation of n -point correlation functions of conformal fields on a genus g Riemann surface. These depend on the positions (z_i, \bar{z}_i) , $i = 1, \dots, n$ of the fields and of the $3g - 3$ complex moduli of the Riemann surface. Due to the factorization of eq. (3), n -point correlation functions are sesquilinear forms on the moduli

space of the punctured Riemann surface [11]

$$W_n = \sum_{I, \bar{I}} g_{I\bar{I}} \mathcal{F}_I \bar{\mathcal{F}}_{\bar{I}} \quad . \quad (5)$$

The analytic blocks \mathcal{F} and $\bar{\mathcal{F}}$ are correlators of CVO and, as such, have non-trivial monodromy and modular properties, even if the W_n is single valued and modular invariant. For instance, in minimal models they correspond to the conformal blocks of BPZ [6] for $g = 0$, $n = 4$, while for $g = 1$, $n = 0$ they are characters of the algebra \mathcal{A} . Another basic feature of RCFT is that correlators of *primary* CVO are a basis of solutions of PDE's obtained from conformal Ward identities with the use of the null vector method [6]. In principle, the knowledge of the chiral observable algebra allows us to construct the chiral correlators by sewing three point functions on the sphere [11] [12] [13], characterized by the OPE coefficients. Alternatively, we can factorize an n -point correlator, by degenerating in some channels the moduli of punctured Riemann surface and using the OPE's. This factorization procedure is not unique and gives rise to the so called *sewing constraints*. In fact, analytic blocks are connected to each other by matrices that represent the action of the Braid Group on the external punctures (B matrices), of the duality transformations (F matrices) and of the modular group generators (T and S in the genus-one case) [7]. It is possible to demonstrate that only two independent sewing constraints occur in closed CFT [13] [7]. They are the crossing symmetry of the four point function on the sphere and the invariance under the “cutting” along the two different homology cycles of the one point function on the torus. More simply, if we limit ourselves to the study of the (perturbative) spectrum of a model, i.e. to the one-loop partition function, all what we need is *modular invariance* [14] [15] [16]. By defining as usual the characters of the algebra \mathcal{A}

$$\chi_i(\tau) = Tr_{(\mathcal{H}_i)} q^{(L_0 - c/24)} \quad , \quad (6)$$

with $q = e^{2\pi i\tau}$, the torus partition function reads

$$T = \sum_{i,j} \chi_i(\tau) N_{ij} \bar{\chi}_j(\bar{\tau}) \quad (7)$$

and must be invariant under the modular group generated by the transformations

$$T : \tau \rightarrow \tau + 1 \quad \text{and} \quad S : \tau \rightarrow -\frac{1}{\tau} \quad , \quad (8)$$

acting on characters χ_i via two matrices, also denoted by T and S . It should be noticed that N_{ij} are 0 or 1 once the ambiguity between characters with the same q -expansion is resolved [17] by carefully extending the modular matrices S and T so that, if C is the charge conjugation matrix,

$$(S)^2 = (ST)^3 = C \quad . \quad (9)$$

Correspondingly, the fusion-rule coefficients N_{ij}^k , connected to the S matrix via the Verlinde formula [18]

$$N_{ij}^k = \sum_n \frac{S_i^n S_j^n S_n^{\dagger k}}{S_0^n} \quad , \quad (10)$$

are also integer and count the number of independent three point functions of primary CVO.

In order to build spectra of closed (super)string models, we have to tensorize chiral sectors of CFT (or their extension to an N=1 superconformal algebra) in such a way that they saturate the *conformal anomaly* [19] and give rise to a modular invariant genus-one partition function consistent with spin-statistics. The total central charge of each chiral sector in the light-cone gauge must then be 12 if the sector is supersymmetric and 24 if it is bosonic. There are thus three classes of interesting strings: bosonic with $c = \bar{c} = 24$, heterotic with $c = 12$ and $\bar{c} = 24$ and type II with $c = \bar{c} = 12$. In particular, in d transverse dimensions, $(3d/2)$ is the contribution to the central charge of space-time supersymmetric coordinates and $(12 - 3d/2)$ is the one of the “internal” theory, while d and $(24 - d)$ are the corresponding values in the bosonic case.

It is worth at this stage to illustrate as simple examples the partition functions of all interesting type II superstrings in ten dimensions ($d = 8$). They are written in terms of characters of the level-one $so(8)$ representations

$$\begin{aligned} O_8 &= \frac{1}{2\eta^4} \left(\theta^4 \begin{bmatrix} 0 \\ 0 \end{bmatrix} + \theta^4 \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} \right) \quad , \quad V_8 = \frac{1}{2\eta^4} \left(\theta^4 \begin{bmatrix} 0 \\ 0 \end{bmatrix} - \theta^4 \begin{bmatrix} 0 \\ 1/2 \end{bmatrix} \right) \quad , \\ S_8 &= \frac{1}{2\eta^4} \left(\theta^4 \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} + \theta^4 \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \right) \quad , \quad C_8 = \frac{1}{2\eta^4} \left(\theta^4 \begin{bmatrix} 1/2 \\ 0 \end{bmatrix} - \theta^4 \begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix} \right) \quad , \end{aligned} \quad (11)$$

with η the Dedekind function and $\theta \begin{bmatrix} \alpha \\ \beta \end{bmatrix}$ the theta functions with characteristics. If we omit the integration over moduli and the fixed contribution of the eight transverse bosonic

coordinates, the left-right symmetric modular invariants are [14] [15] [20]

$$T_{IIB} = |V_8 - S_8|^2 \quad , \quad (12)$$

$$T_{0A} = |O_8|^2 + |V_8|^2 + S_8 \bar{C}_8 + C_8 \bar{S}_8 \quad , \quad (13)$$

$$T_{0B} = |O_8|^2 + |V_8|^2 + |S_8|^2 + |C_8|^2 \quad . \quad (14)$$

The first is the “Type IIB” superstring while the other two, being tachyonic and not supersymmetric, have to be considered as toy models but are interesting for the open-string program. The other (not left-right symmetric) well known model in ten dimension is the “Type IIA” superstring, whose partition function results

$$T_{IIA} = (V_8 - S_8)(\bar{V}_8 - \bar{C}_8) \quad . \quad (15)$$

3. Open and unoriented CFT and superstrings

The basic building blocks of open (super)strings are CFT defined on arbitrary Riemann surfaces. This amount to take into account, in addition to theories on closed orientable Riemann surfaces, also theories propagating on surfaces with boundaries and/or crosscaps, three crosscaps being equivalent to one handle and one crosscap [21]. This extended geometrical framework makes necessary the inclusion of additional data in order to properly define the CFT. First of all, the introduction of boundary (or open) fields ψ_i^{ab} beside the bulk conformal fields (3) is required [22] [23]. ψ_i^{ab} lives exactly on boundaries and its insertion changes type a boundary conditions into type b ones. The crucial observation is that each Riemann surface with holes and/or crosscaps admits a closed orientable double cover and can be thus defined as the quotient of the double cover by an (anticonformal) involution [24]. As a consequence, the two holomorphic and antiholomorphic chiral algebras are no longer independent. On the one hand, this implies the existence of a chiral algebra that contains the “diagonal” combination of holomorphic and antiholomorphic Virasoro algebras [23]. On the other hand, n -point correlation functions in the presence of holes and/or crosscaps become real linear rather than sesquilinear combinations of analytic blocks. The presence of open fields makes also necessary the introduction of two

more OPE's [25] [26]. One is the product of two boundary operators, expressible in terms of the three point functions on the disk C_{ijk}^{abc} . The other concerns the behaviour of a bulk field $\phi_{k,\bar{k}}$ approaching a boundary. What happens is that the bulk field “collides” with its image and can be expanded in terms of only boundary operators. The corresponding OPE coefficients $C_{(k,\bar{k})i}^a$ are related to the amplitude on the disk with an “open” and a “closed” puncture. Moreover, the algebra of boundary fields acts on the same Hilbert space as the (chiral part of) the algebra of the bulk fields. As a result, the field normalization is in general diverse and other coefficients are needed to complete the definition of the CFT. To be precise, there is the necessity of coefficients α_i^{ab} [25] for taking into account the normalization of the two-point function of boundary fields and coefficients Γ_k [27] responsible for the normalization of the one-point function of bulk fields $\phi_{(k,\bar{k})}$ in front of a crosscap. This one-point function is essentially a chiral two-point function on the sphere of a CVO with its image under the involution, and can be calculated as a vacuum amplitude with an insertion of the “crosscap operator” [28]

$$\hat{C} = \sum_k \Gamma_k |\Delta_k \rangle \langle \bar{\Delta}_k| \quad , \quad (16)$$

where $|\Delta_k \rangle$ corresponds to the primary CVO V_k . In particular

$$\langle \phi_{1,\bar{1}} \dots \phi_{n,\bar{n}} \rangle_C = \langle \hat{C} \phi_{1,\bar{1}} \dots \phi_{n,\bar{n}} \rangle_0 \quad , \quad (17)$$

and, by eq. (16), the last expression is a chiral $2n$ -point correlation function. It should be noticed that, due to the non-trivial topology of the crosscap, a bulk field crossing a crosscap can emerge identical or opposite to its image. Denoting this sign with ε and indicating with X any polynomial in the fields, one finds

$$\langle \phi_{k,\bar{k}}(z_k, \bar{z}_k) X \rangle_C = \varepsilon_{(k,\bar{k})} \langle \phi_{\bar{k},k}(\bar{z}_k, z_k) X \rangle_C \quad , \quad (18)$$

where the image field is obtained from the original field by interchanging holomorphic and antiholomorphic parts. Finally, the normalization coefficients B_k^a of the one-point function of $\phi_{(k,\bar{k})}$ in front of a boundary, also necessary, are a subset of the bulk-boundary OPE coefficients, namely $B_k^a = C_{(k,\bar{k})1}^a \alpha_1^{aa}$.

In principle again, we are able to build every correlation function of open and bulk fields on arbitrary Riemann surfaces by sewing the three building blocks corresponding

to the OPE previously introduced and taking into account the normalization coefficients. Making the “sewing procedure” unambiguous produces again a finite number of sewing constraints. The analysis of all sewing constraints is beyond the scope of this talk. The interested reader can refer to the original literature [13] [7] [25] [27] [28]. Rather, it is worth discussing one of them, the “*crosscap constraint*” introduced in refs. [27] [28], that determines the whole spectrum in the non-orientable closed sector and can be exactly solved for large classes of models like, for instance, the infinite series of A-D-E minimal models and $SU(2)$ WZW models of CIZ [29]. As previously mentioned, the crosscap can be defined by identifying, via an anticonformal involution, opposite points on the Riemann sphere. As a result, only an $SU(2)$ subgroup of the (global) conformal group $SL(2, \mathbb{C})$ “descends” to the crosscap. Actually, this residual symmetry encoded in the one-point function on the crosscap already constraints the values of Γ_k . Indeed, it is easy to show that the one-point function of a field coincide with the one-point function of the image:

$$\langle \phi_{k,\bar{k}}(z, \bar{z}) \rangle_C = \Gamma_k \delta_{\Delta_k, \bar{\Delta}_k} \langle 0 | V_k(z) V_{\bar{k}}(\bar{z}) | 0 \rangle = \langle \phi_{\bar{k},k}(\bar{z}, z) \rangle_C \quad . \quad (19)$$

As a consequence, Γ_k is vanishing for fields with non-zero spin ($\Delta_k \neq \bar{\Delta}_k$) or with $\varepsilon = -1$, with ε the phase in eq. (18). The two-point function in front of a crosscap is the relevant amplitude for the “crosscap constraint”. In deriving it, there is in fact an ambiguity in inserting the crosscap state of eq. (16). Indeed, \hat{C} can be put between the punctures 1 and 2 and the two images $\bar{1}$ and $\bar{2}$, else it can separate punctures 1 and $\bar{2}$ from the remaining $\bar{1}$ and 2 (Fig 1).

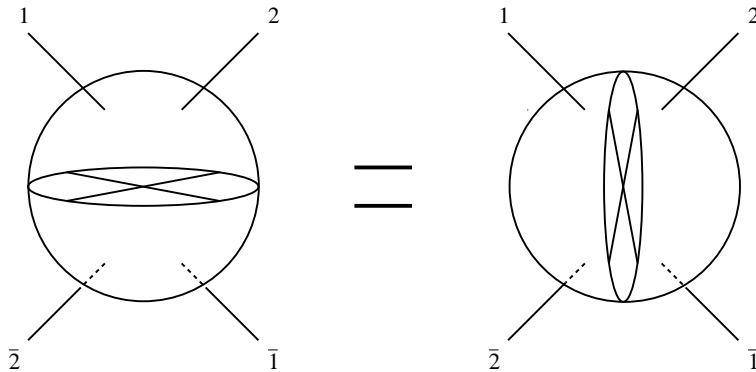


Figure 1. The “crosscap constraint”

Technically, using the fusion matrices, we can relate the two-point function

$$\langle \phi_{(1,\bar{1})}(z_1, \bar{z}_1) \phi_{(2,\bar{2})}(z_2, \bar{z}_2) \rangle_C = \sum_k \Gamma_k C_{(1,\bar{1})(2,\bar{2})}^{(k,k)} S_k(z_1, z_2, \bar{z}_1, \bar{z}_2) \quad (20)$$

to the two point function

$$\langle \phi_{(\bar{1},1)}(\bar{z}_1, z_1) \phi_{(2,\bar{2})}(z_2, \bar{z}_2) \rangle_C = \sum_k \Gamma_k C_{(\bar{1},1)(2,\bar{2})}^{(k,k)} S_k(\bar{z}_1, z_1, z_2, \bar{z}_2) \quad , \quad (21)$$

where S_k is the s -channel conformal block with a k field in the intermediate state and the C 's are the closed OPE coefficients. Indeed, the following relation holds

$$S_k(\bar{z}_1, z_2, z_1, \bar{z}_2) = (-1)^{\Delta_1 - \bar{\Delta}_1 + \Delta_2 - \bar{\Delta}_2} \sum_n F_{kn}(1, 2, \bar{1}, \bar{2}) S_n(z_1, z_2, \bar{z}_1, \bar{z}_2) \quad , \quad (22)$$

where the phase comes from a braiding $B_1(B_3)^{-1}$ and F 's are the fusion matrices. Inserting this expression in eq. (21) and using eq.(18) we obtain the “crosscap constraint”, a linear relation between the one-point coefficients Γ_k

$$\varepsilon_{(1,\bar{1})} (-1)^{\Delta_1 - \bar{\Delta}_1 + \Delta_2 - \bar{\Delta}_2} \Gamma_n C_{(1,\bar{1})(2,\bar{2})}^{(n,n)} = \sum_k \Gamma_k C_{(\bar{1},1)(2,\bar{2})}^{(k,k)} F_{kn}(1, 2, \bar{1}, \bar{2}) \quad . \quad (23)$$

Eq. (23) plays a fundamental role also in discussing rules to construct perturbative spectra of open and unoriented (super)strings, fully encoded in the one-loop partition function. To tackle the whole construction from this equivalent point of view, two preliminary observations are needed. First, it has long been known that a theory of only boundary operators is inconsistent, the reason being that bulk fields are always present in the intermediate states of non-planar open diagrams [30]. Second, enclosing the non orientable contribution, i.e. allowing a “twist” [31] of strings, is demanded, as we will see, by the structure of ultraviolet divergencies and of anomaly cancellations. Four contributions enter the one-loop partition function. The starting one is the torus contribution of eq. (7) that encodes the spectrum of the closed oriented “parent” model. In order to construct a class of “open descendants”, we have to project the closed spectrum to a non-orientable one. This is obtained adding to the (halved) torus the (halved) Klein bottle amplitude. Then we have to add the two (halved) open contributions, the annulus and Möbius strip amplitudes, that describe the open unoriented spectrum. The construction is very reminiscent of what happens in the Z_2 orbifolds [32], where the closed spectrum is projected

in a Z_2 -invariant way and “twisted” sectors corresponding to strings closed only on the orbifold are added and projected. Since open strings are in some sense closed on the double cover, the orbifold should be thought in the parameter space rather than in the target space [5] [33]. The open states are then analog to “twisted sectors”, while the role of Z_2 group is played by the twist that interchanges left (holomorphic) and right (antiholomorphic) sector. This is the reason why *only left-right symmetric “parent” closed models can admit a class of open descendants*. The amplitude in eq. (7) must thus refer to models with identical holomorphic and antiholomorphic sectors (the Type IIA superstring, for instance, does not admit open descendants with 10-D Lorentz symmetry).

To understand better the consistency conditions, let us take a closer look at the three additional amplitudes. The (direct channel) Klein bottle amplitude, that projects the closed spectrum of eq. (7), has the general form

$$K = \frac{1}{2} \sum_i \chi_i K_i \quad (24)$$

with $|K_i| = N_{ii}$. Actually, more choices are possible according to the signs of K_i , that determine the (anti)symmetrization properties of the “ i ” sector. The (anti)symmetry of Verma modules must be compatible with the fusion rules. For instance, the fusion of two states in antisymmetrized sectors must produce symmetric states. In particular, the available choices corresponds to the Z_2 -automorphism of the fusion algebra compatible with the torus partition function (see also ref. [34] for an alternative derivation based on Chern-Simons theory on orbifolds). Performing an S modular transformation on K , we turn to the “transverse channel” Klein bottle amplitude that describes the propagation of the closed spectrum between two crosscaps states (Fig. 2).

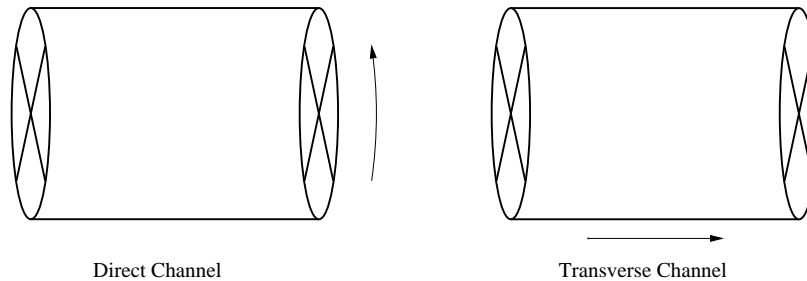


Figure 2. The Klein Bottle

It has the general form

$$\tilde{K} = \frac{1}{2} \sum_i \chi_i (\Gamma_i)^2 \quad , \quad (25)$$

where Γ_i are the one-point coefficients of eq (16). As previously stated, they are completely determined by solving the “crosscap constraint”. Notice that coefficients in \tilde{K} are perfect squares, while the gammas are directly related to the phase choices in K and are consistent with the fusion rules.

Let us now come to the description of the open spectrum, the genuine new ingredient of these models. It has been known for a long time that open-string ends carry (Chan-Paton) charges [35] [36] that manifest themselves, at the level of partition functions, as multiplicities to be associated to the annulus and Möbius amplitudes. After all, in conventional orbifolds multiplicities are associated to fixed points, and boundaries are fixed under the involution. Let us start from the transverse annulus amplitude that describes the closed spectrum flowing between two boundary states (Fig. 3).

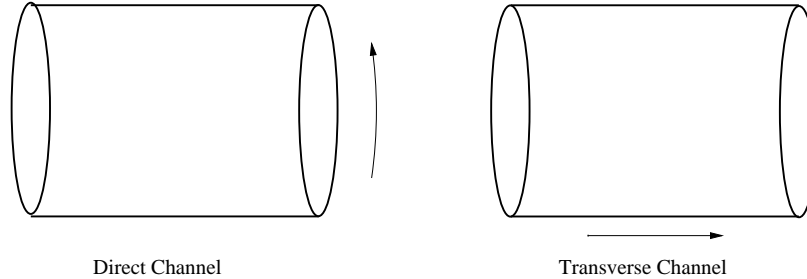


Figure 3. The Annulus

Due to the mirror-like properties of the boundary, *only fields which are paired with the conjugates in the closed GSO projection of eq. (7) can appear in the transverse annulus* [37]. Technically, they are the only states with non-vanishing B_k^a coefficients. If we introduce the Chan-Paton multiplicities n^a , the transverse annulus amplitude can be written

$$\tilde{A} = \frac{1}{2} \sum_k \chi_k \left(\sum_a B_k^a n^a \right)^2 \quad , \quad (26)$$

where the perfect squares indicate, as expected, the presence of two boundaries in the annulus. Performing as in the Klein bottle amplitude the S modular transformation

exposes the direct-channel annulus amplitude, that reads

$$A = \frac{1}{2} \sum_{k,a,b} A_{ab}^k n^a n^b \chi_k \quad . \quad (27)$$

The non negative integer coefficients A_{ab}^k are very important, because they determine the open spectrum or, equivalently, classify the set of conformally invariant boundary conditions. The interpretation of eq. (27) is that open states of the k sector with charges n^a and n^b and multiplicities corresponding to A_{ab}^k can exist if A_{ab}^k itself is non-vanishing. In the *diagonal* RCFT, the flow of the k -th sector along a strip with boundary condition a and b is governed by the fusion-rule coefficients [22]. This implies that $A_{ab}^k = N_{ab}^k$ (diagonal ansatz), and using eq. (10) allows to express the B_k^a in terms of entries of the modular matrix S [22] [26]:

$$B_k^a = \frac{S_k^a}{\sqrt{S_k^1}} \quad . \quad (28)$$

In the more complicated non-diagonal models, B_k^a are determined by solving suitable sewing constraints and A_{ab}^k are no-longer the fusion-rule coefficients [28]. Rather, they satisfy completeness relations and can manifest the presence of extended boundary operator algebras, as compared to the diagonal case [38]. Finally, the direct channel Möbius amplitude (anti)symmetrizes the open spectrum, and has the form

$$M = \pm \frac{1}{2} \sum_{k,a} M_a^k n^a \hat{\chi}_k \quad , \quad (29)$$

where $M_a^k = A_{aa}^k \pmod{2}$ and $\hat{\chi}_k$ are the suitable characters that count states on the Möbius strip, whose modulus is not purely imaginary. The overall sign is free in RCFT, but is crucial in critical open-string models where it determines the type of gauge (Chan-Paton) groups. The transverse channel Möbius contributions that represent closed states flowing between a hole and a crosscap (Fig. 4)

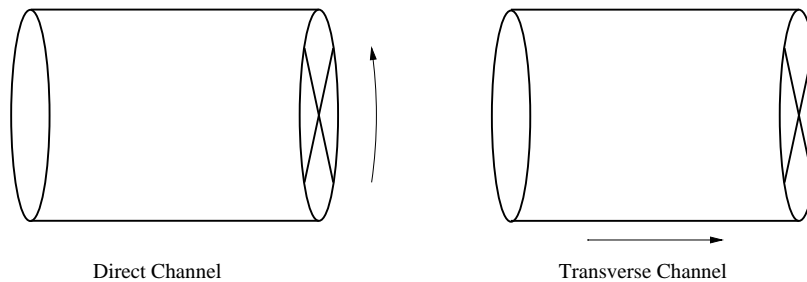


Figure 4. The Moebius Strip

are, as intuitively expected, geometric means of those in \tilde{A} and \tilde{K} :

$$\tilde{M} = \pm \sum_k \left(\sum_a B_k^a n^a \right) \Gamma_k \hat{\chi}_k \quad . \quad (30)$$

In order to obtain M from \tilde{M} and thus determine the signs in M_k^a the matrix $P = T^{1/2} S T^2 S T^{1/2}$ must be used that implements on the basis of “hatted” characters the modular transformation $\frac{i\tau+1}{2} \rightarrow \frac{i+\tau}{2\tau}$. The compatibility between M and \tilde{M} is the last constraint. It should be stressed that given a left-right symmetric closed oriented model, the algorithm allows to build a class of consistent open descendants. In particular, open amplitudes automatically satisfy planar duality and factorization properties [39] [40], as we shall see in an example in section 4. Two final observations are in order: first, open states can be, in some sectors, oriented. This reflects itself in the presence of complex Chan-Paton charges [37]. Second, in critical (super)string models the transverse amplitudes \tilde{K} , \tilde{A} and \tilde{M} exhibit the flow of massless scalars that can acquire a VEV (tadpoles) and in principle have to be eliminated from the spectrum. This requires the cooperative action of all transverse channel amplitudes and justifies the introduction of the non-orientable contributions [36] [41]. While the presence of tadpoles of “physical” scalars signal a vacuum instability that could hide a Higgs-like phenomenon, tadpoles of “unphysical” scalars produce inconsistencies and must be eliminated. It is possible to show that the cancellation of tadpoles of “unphysical” scalars is equivalent to the cancellation of all gauge and gravitational anomalies in the low energy effective field theory [42].

4. Examples

In order to appreciate the algorithm described in the preceding section, it is useful to analyze some concrete examples. In particular, let us start by deriving the open descendants of the Ising model, the simplest model in the infinite series of minimal $A - A$ modular invariants, together with the descendants of A_3 model, the simplest non trivial model in the A -series of $SU(2)$ WZW modular invariants [40]. Both are diagonal and the open descendants are based, as said, on the diagonal ansatz. However, the presence of the $SU(2)$ Kac-Moody algebra in the A_3 case reflects itself in a number of subtleties that are

worthy of a detailed discussion and are common to all models in the two infinite series.

The Ising and A_3 models on the torus share formally the same partition function

$$T = |\chi_1|^2 + |\chi_2|^2 + |\chi_3|^2 \quad , \quad (31)$$

provided one identifies χ_1 , χ_2 and χ_3 with the characters of identity, spin and energy for Ising, and with χ_{2I+1} , with I the isospin, for A_3 . The S matrix is identical as well,

$$S = \frac{1}{2} \begin{pmatrix} 1 & \sqrt{2} & 1 \\ \sqrt{2} & 0 & -\sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix} \quad , \quad (32)$$

but open descendants must be different because, due to the fact that the conformal dimension of χ_2 is $3/16$ while that of the Ising spin is $1/16$, their P matrices are

$$P_{(Is)} = \begin{pmatrix} c & 0 & s \\ 0 & 1 & 0 \\ s & 0 & -c \end{pmatrix} \quad , \quad P_{(A_3)} = \begin{pmatrix} s & 0 & c \\ 0 & 1 & 0 \\ c & 0 & -s \end{pmatrix} \quad , \quad (33)$$

with $s = \sin(\pi/8)$ and $c = \cos(\pi/8)$. There are two possible Klein bottle projections, in correspondence with the only possible automorphism of the fusion rules, the Z_2 center of $SU(2)$ in A_3 and the spin reversal in the Ising model:

$$K_1 = \frac{1}{2} (\chi_1 + \chi_2 + \chi_3) \quad , \quad K_2 = \frac{1}{2} (\chi_1 - \chi_2 + \chi_3) \quad . \quad (34)$$

It should be noticed, however, that K_1 corresponds to having the Ising spin symmetric in front of the crosscap (i.e. the ε of eq. (18) equal to $+1$) and K_2 to having the Ising spin antisymmetric ($\varepsilon = -1$). On the contrary, K_1 corresponds to having the isospin $1/2$ field of A_3 antisymmetric ($\varepsilon = -1$) in front of the crosscap and K_2 to having it symmetric ($\varepsilon = +1$). The open spectrum feels the signs, because they enter the “crosscap constraint” and thus the coefficients Γ_k , that appear in \tilde{K} and in \tilde{M} . As a consequence, we have four different classes of open descendants. K_1 leads to descendants with real Chan-Paton charges for Ising

$$\begin{aligned} A1_{(Is)} &= \left(\frac{n_0^2 + n_{1/2}^2 + n_{1/16}^2}{2} \right) \chi_0 + n_{1/16} (n_0 + n_{1/2}) \chi_{1/16} + \left(\frac{n_{1/16}^2}{2} + n_0 n_{1/2} \right) \chi_{1/2} , \\ M1_{(Is)} &= \pm \left[\frac{n_0 + n_{1/16} + n_{1/2}}{2} \hat{\chi}_0 + \frac{n_{1/16}}{2} \hat{\chi}_{1/2} \right] \quad , \end{aligned} \quad (35)$$

and with complex Chan-Paton charges for A_3

$$\begin{aligned} A1_{(A_3)} &= \left(\frac{n_2^2}{2} + m\bar{m} \right) \chi_1 + n_2(m + \bar{m}) \chi_2 + \frac{n_2^2 + m^2 + \bar{m}^2}{2} \chi_3, \\ M1_{(A_3)} &= \pm \left[\frac{n_2}{2} \hat{\chi}_1 + \frac{n_2 + m + \bar{m}}{2} \hat{\chi}_3 \right] . \end{aligned} \quad (36)$$

The opposite is true for K_2 , that leads to descendants with complex Chan-Paton charges for Ising

$$\begin{aligned} A2_{(Is)} &= \left(\frac{n_{1/16}^2}{2} + m\bar{m} \right) \chi_0 + n_{1/16}(m + \bar{m}) \chi_{1/16} + \frac{n_{1/16}^2 + m^2 + \bar{m}^2}{2} \chi_{1/2}, \\ M2_{(Is)} &= \pm \left[\frac{n_{1/16}}{2} \hat{\chi}_0 + \frac{m + \bar{m} - n_{1/16}}{2} \hat{\chi}_{1/2} \right] , \end{aligned} \quad (37)$$

and to descendants with real Chan-Paton charges for A_3

$$\begin{aligned} A2_{(A_3)} &= \left(\frac{n_1^2 + n_2^2 + n_3^2}{2} \right) \chi_1 + n_2(n_1 + n_3) \chi_2 + \left(\frac{n_2^2}{2} + n_1 n_3 \right) \chi_3, \\ M2_{(A_3)} &= \pm \left[\frac{n_1 - n_2 + n_3}{2} \hat{\chi}_1 + \frac{n_2}{2} \hat{\chi}_3 \right] . \end{aligned} \quad (38)$$

This structure repeats itself for the whole $A-A$ series of minimal models and for the whole A -series of $SU(2)$ WZW models. Given a modular invariant torus partition function, there exist two classes of open descendants with real or complex charges. For instance, the modular invariant of the A series at level k , in the same notation of eq. (31), is the diagonal one

$$T = \sum_{a=1}^{k+1} |\chi_a|^2 , \quad (39)$$

and the Klein bottle projection leading to all real charges is

$$K = \frac{1}{2} \sum_{a=1}^{k+1} (-1)^{(a-1)} \chi_a . \quad (40)$$

The annulus amplitude is directly obtainable by the diagonal ansatz, while the Möbius amplitude exhibits signs reflecting the underlying current algebra

$$\begin{aligned} A &= \frac{1}{2} \sum_{a,b,c} N_{ab}^c n^a n^b \chi_c , \\ M &= \pm \frac{1}{2} \sum_{a,b} (-1)^{b-1} (-1)^{\frac{a-1}{2}} N_{bb}^a n^b \hat{\chi}_a . \end{aligned} \quad (41)$$

In particular, as discussed for A_3 , the phase $(-1)^{\frac{a-1}{2}}$ is due to the behaviour of fields in front of the crosscap and the phase $(-1)^{b-1}$ changes the type of charges in a way

corresponding to the isospin. Moreover, the coefficients Γ_k can nicely be expressed in terms of S and P matrices

$$\Gamma_k = \frac{P_{1k}}{\sqrt{S_{1k}}} \quad , \quad (42)$$

and allow direct channel K and M amplitudes to be written using some components of the integer-valued tensor [40]

$$Y_{abc} = \sum_d \frac{S_{ad} P_{bd} P_{cd}^\dagger}{S_{1d}} \quad . \quad (43)$$

It is also interesting to briefly mention how *relative* signs in the non-orientable contributions are connected to the action of “twist” respecting factorization and planar duality of amplitudes [39] [40]. To this end, it is sufficient to consider the Ising model.

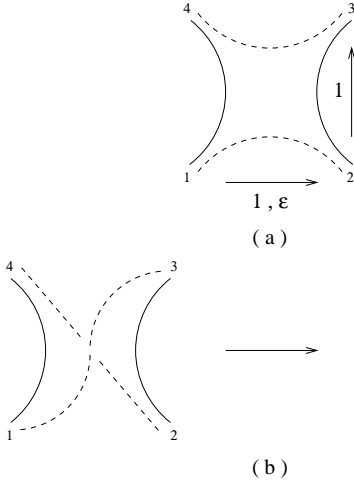


Figure 5. “Twist” with real charges.

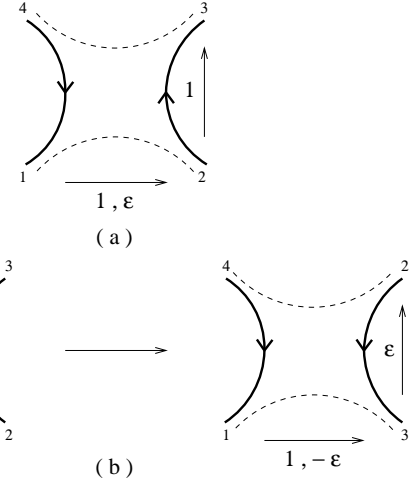


Figure 6. “Twist” with complex charges.

Referring to figure 5, let us first analyze the four-spin amplitude on the disk for the model with real charges. Due to the fusion rules, indicating with dashed lines $n_{1/16}$ charges and with continuous lines n_0 charges, only the identity flows in the s -channel, while identity and energy flow in the u -channel (Fig. 5a). If we take into account the “twisted” u -channel amplitude, we can see that it is related, by an operation of “unfolding”, to an amplitude in which the identity flows in the t -channel and again identity and energy flow in the u -channel with the same sign. This means that identity and energy with a pair of $n_{1/16}$ charges “twist” in the same way, as demanded by $A1_{(Is)}$ and $M1_{(Is)}$ in eq. (35). Let us then consider the same four-spin amplitude, but referred to the model with complex

charges (Fig. 6). If dashed lines indicate again $n_{1/16}$ charges, we have now to introduce arrows associated to complex charges m and \bar{m} . An inspection of the partition function reveals that in the s -channel only the identity with charges m and \bar{m} can flow (fig 6a). By fusion, again the u channel “sees” identity and energy with a “pair” of $n_{1/16}$ charges. However, u -channel “twist” is now no longer “ineffective” on charges, since the unfolding inverts the relative orientation of arrows. As a result, the twisted amplitude admits only the energy flowing in its t -channel and consequently identity and energy in its u -channel, but with *opposite* signs, as demanded by $A2_{(Is)}$ and $M2_{(Is)}$ in eq. (37). The opposite sign in the behaviour under twist of χ_1 and χ_3 in the A_3 model with respect to the identity and energy of Ising may be traced to the opposite sign in the behaviour of the corresponding conformal blocks under the action of the fusion matrix F [40].

Open descendants of modular invariants of D and E type can be constructed as well. In particular, E_{even} and D_{even} models exhibit an extended algebra and become (quasi)diagonal after the resolution of ambiguity. This introduces factors of two in some fusion rules, corresponding to the possibility of having some multiple families with same charges and also multiple three point open functions. The off-diagonal models E_7 and D_{odd} are less conventional because, as said in section 3, they are not directly based on the fusion rules, and a number of subtleties arise from boundary operators and related sewing constraints [38].

Let us conclude by displaying some examples concerning open (super)strings. There is only one open descendant of the Type II superstring in $D = 10$, the long known Type I $SO(32)$ superstring [43]. Indeed, the closed spectrum exhibits a single chiral “supercharacter” corresponding to the GSO projection, namely $V_8 - S_8$. It flows also on the annulus and is modular invariant by itself. Only one Chan-Paton charge is present and the direct channel contributions are

$$K_{IIB} = \frac{1}{2} (V_8 - S_8) \quad , \quad (44)$$

$$A_{IIB} = \frac{1}{2} n^2 (V_8 - S_8) \quad , \quad (45)$$

$$M_{IIB} = \pm \frac{1}{2} n (V_8 - S_8) \quad . \quad (46)$$

Performing the modular transformations to turn the amplitudes into the transverse channel, crucial factors of two arise from the (omitted) modular measure due to the fact that a comparison of different surfaces is possible only in terms of the double cover [5] [44]. To be precise, \tilde{K} gains a factor 2^5 while \tilde{A} gains a factor 2^{-5} . The cancellation of the “unphysical” massless scalar of the R-R sector forces the Möbius overall sign to be negative and the coefficient in front of S_8 to be zero:

$$\frac{2^5}{2} + \frac{n^2 2^{-5}}{2} - n = 0 \quad . \quad (47)$$

The resulting model is precisely the type I $SO(32)$ superstring of Green and Schwarz. There exists also a bosonic analog of this model with gauge group $SO(8192)$ [36] [41].

Open descendants of Type 0 models are very instructive since they exhibit many aspects of the general construction [37]. Let us concentrate on the type 0B model, in which four Chan-Paton charge sectors are present due to the (self)conjugation properties of the characters (11). Notice that only two charges corresponding to the two characters flowing in \tilde{A} would enter the Type 0A descendants. There are three possible inequivalent choices of Klein bottle projection compatible with fusion rules and positivity of the transverse channel:

$$K_{0B} = \frac{1}{2} (O_8 + V_8 - S_8 - C_8) \quad , \quad (48)$$

$$K'_{0B} = \frac{1}{2} (O_8 + V_8 + S_8 + C_8) \quad , \quad (49)$$

$$K''_{0B} = \frac{1}{2} (V_8 - O_8 + S_8 - C_8) \quad . \quad (50)$$

The first one is referred to the conventional choice of a basis in the light cone gauge of $SO(1,9)$, where the NS-NS sector is symmetrized while the R-R sector antisymmetrized. The other two correspond to the basis (O_8, V_8, S_8, C_8) and $(-O_8, V_8, -S_8, C_8)$. It should be stressed that, while the closed spectrum surviving the K'' projection does not contain tachyons and is chiral, both K and K' leave tachyons. Open sectors can be constructed directly in terms of the diagonal ansatz, using the general prescription of eq. (27) with $A_{ab}^k = N_{ab}^k$. However, in the transverse channel of Klein bottle amplitudes (and, consequently, in the transverse channel of Möbius strip amplitudes) only one character flows:

$$\tilde{K}_{0B} = \frac{2^6}{2} V_8 \quad , \quad (51)$$

$$\tilde{K}'_{0B} = \frac{2^6}{2} O_8 \quad , \quad (52)$$

$$\tilde{K}''_{0B} = -\frac{2^6}{2} C_8 \quad , \quad (53)$$

As a result, even in the direct channel Möbius amplitude a single character is present, the same as in the transverse Klein bottle amplitude, since the P matrix is diagonal in this case. The character V_8 plays the role of identity in the light-cone gauge of $SO(1,9)$. This implies that an assignment of *real* Chan-Paton charges is compatible only with the K_{0B} projection, and results in

$$\begin{aligned} A_{0B} &= \frac{n_o^2 + n_v^2 + n_s^2 + n_c^2}{2} V_8 + (n_o n_v + n_s n_c) O_8 \\ &\quad - (n_o n_c + n_v n_s) S_8 - (n_o n_s + n_v n_c) C_8 \quad , \end{aligned} \quad (54)$$

$$M_{0B} = -\frac{1}{2} (n_o + n_v + n_s + n_c) \hat{V}_8 \quad . \quad (55)$$

It should be noticed that the open spectrum is chiral. An inspection of the transverse channel reveals three tadpole conditions. Two of them, corresponding to R-R “unphysical” scalars, give $n_o = n_v$ and $n_s = n_c$ and guarantee the cancellation of gauge and gravitational anomalies. The third one, “physical”, is the tadpole relative to the dilaton. If we cancel it, the total Chan-Paton group dimensionality is fixed to 64, and the open sector exhibits an $SO(n) \otimes SO(32-n) \otimes SO(n) \otimes SO(32-n)$ symmetry group.

The other two projections are only compatible with complex Chan-Paton charges. Looking, for instance, to the more interesting model of eq. (50), the charge assignment must be the following [45]:

$$\begin{aligned} A''_{0B} &= (n_1 \bar{n}_1 + n_2 \bar{n}_2) V_8 + (n_1 \bar{n}_2 + n_2 \bar{n}_1) O_8 \\ &\quad - (n_1 n_2 + \bar{n}_1 \bar{n}_2) S_8 - \left(\frac{n_1^2}{2} + \frac{n_2^2}{2} + \frac{\bar{n}_1^2}{2} + \frac{\bar{n}_2^2}{2} \right) C_8 \quad , \end{aligned} \quad (56)$$

$$M''_{0B} = -\frac{1}{2} (n_1 + \bar{n}_1 - n_2 - \bar{n}_2) \hat{C}_8 \quad . \quad (57)$$

The gauge vector is not projected on the Möbius strip because, having ends carrying a “quark-antiquark” pair of $U(n_1) \otimes U(n_2)$, is oriented. The (numerical) equalities $n_1 = \bar{n}_1$ and $n_2 = \bar{n}_2$ are necessary in order to avoid negative reflection coefficients in front of the boundaries. Thus, only one “unphysical” tadpole condition survives, giving the constraint

$n_1 - n_2 = 32$. The dilaton tadpole cannot be canceled in this model, but this, rather than being a problem, could open new perspectives in the analysis of low energy models [46]. Large classes of open models based on superstrings built with the free-fermion construction [47] can be consistently defined in lower dimension [37]. In particular, chiral models in $D = 6$ are anomaly free thanks to a generalized Green-Schwarz cancellation mechanism [48]. Chiral four-dimensional models can also be built, but models so far analyzed exhibit small-sized gauge groups. It is our opinion that this is due to the too simple structure of rational closed models built in terms of free-fermions alone. Descendants of models with genuinely interacting CFT in the internal sector, like for instance $N = 2$ models [49], could open new possibilities.

In conclusion, we have described how to consistently define CFT on Riemann surfaces with holes and/or crosscaps. They constitute the basic building blocks of open and unoriented (super)strings. In particular, we have reviewed the role of sewing constraints and the structure of one-loop partition functions, giving some explicit examples of classes of open descendants of left-right symmetric closed oriented models.

Acknowledgments

I am grateful to the organizers for the kind invitation and to M. Bianchi, A. Sagnotti and Ya.S. Stanev for the collaboration on related works. This work was supported in part by E.E.C. Grant CHRX-CT93-0340.

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